MENIIT
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# JEE Advanced : Paper-2 (2017)

## IMPORTANT INSTRUCTIONS

**GENERAL**

1. This sealed booklet is your Question Paper. Do not break the seal till you are told to do so.
2. The paper CODE is printed on the right hand top corner of this sheet and the right hand top corner of the back cover of this booklet.
3. Use the Optical Response Sheet (ORS) provided separately for answering the questions.
4. The paper CODE is printed on the left part as well as the right part of the ORS. Ensure that both these codes are identical and same as that on the question paper booklet. If not, contact the invigilator for change of ORS.
5. Blank spaces are provided within this booklet for rough work.
6. Write your name, roll number and sign in the space provided on the back cover of this booklet.
7. After breaking the seal of the booklet at 9:00 am, verify that the booklet contains 36 pages and that all the 54 questions along with the options are legible. If not, contact the invigilator for replacement of the booklet.
8. You are allowed to take away the Question Paper at the end of the examination.

**OPTICAL RESPONSE SHEET**

9. The ORS (top sheet) will be provided with an attached Candidate's Sheet (bottom sheet). TheCandidate's Sheet is a carbonless copy of the ORS.
10. Darken the appropriate bubbles on the ORS by applying sufficient pressure. This will leave an impression at the corresponding place on the Candidate's Sheet.
11. The ORS will be collected by the invigilator at the end of the examination.
12. You will be allowed to take away the Candidate's Sheet at the end of the examination.
13. Do not tamper with or mutilate the ORS. Do not use the ORS for rough work.
14. Write your name, roll number and code of the examination center, and sign with pen is the space provided for this purpose on the ORS. Do not write any of these details anywhere else on the ORS. Darken the appropriate bubble under each digit of your roll number.

**DARKENING THE BUBBLES ON THE ORS**

15. Use a BLACK BALL POINT PEN to darken the bubbles on the ORS.
16. Darken the bubble COMPLETELY.
17. The correct way of darkening a bubble is as:
18. The ORS is machine-gradable. Ensure that the bubbles are darkened in the correct way.
19. Darken the bubbles ONLY IF you are sure of the answer. There is NO WAY to erase or "un-darken" a darkened bubble. Please

# PART A: PHYSICS

## SECTION 1 (Maximum Marks: 21)

- This section contains SEVEN questions
- Each question has FOUR options [A], [B], [C] and [D]. ONLY ONE of these four options is correct
- For each question, darken the bubble corresponding to the correct option in the ORS.
- For each question, marks will be awarded in one of the following categories :

| | | |
|---|---|---|
| Full Marks | : +3 | If only the bubble corresponding to the correct option is darkened. |
| Zero Marks | : 0 | If none of the bubbles is darkened. |
| Negative Marks | : –1 | In all other cases. |

**1.** A photoelectric material having work - function $\phi_0$ is illuminated with light of wavelength $\lambda\left(\lambda < \frac{hc}{\varphi_0}\right)$. The fastest photoelectron has a de Broglie wavelength $\lambda_d$. A change in wavelength of the incident light by $\Delta\lambda$ results in a change $\Delta\lambda_d$ in $\lambda_d$. Then the ratio $\Delta\lambda_d/\Delta\lambda$ is proportional to

(A) $\lambda_d^3/\lambda^2$ (B) $\lambda_d^2/\lambda^2$ (C) $\lambda_d/\lambda$ (D) $\lambda_d^3/\lambda$

**Ans.** A

**Sol.** $E = \phi + K.E.$

$$\frac{hc}{\lambda} = \phi + \frac{h^2}{2m\lambda_d^2}$$

$$\ell n\frac{hc}{\lambda} = \ell n\left(\varphi + \frac{h^2}{2m\lambda_d^2}\right)$$

$$-\frac{1}{hc/\alpha}\cdot\frac{hcd\lambda}{\lambda^2} = \frac{1}{Q + \frac{h^2}{2m\lambda_d^2}}\left(\frac{h^2}{2m}\frac{d\lambda_d}{\lambda_d^3}\right)(-2)$$

$$\frac{d\lambda}{\lambda} = \frac{\lambda}{hc}\left(\frac{2h^2}{2m}\right)\frac{d\lambda_d}{\lambda_d^3}$$

$$\frac{d\lambda_d}{d\lambda} \propto \frac{\lambda_d^3}{\lambda^2}$$

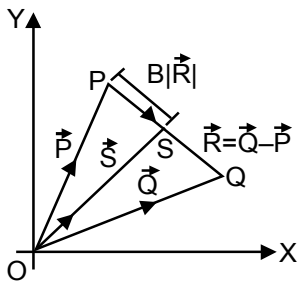
**2.** Three vectors $\vec{P},\vec{Q}$ and $\vec{R}$ are shown in the figure. Let S be any point on the vector $\vec{R}$. The distance between the points P and S is $b|\vec{R}|$. The general relation among vectors $\vec{P},\vec{Q}$ and $\vec{S}$ is

(A) $\vec{S} = (1-b)\vec{P} + b^2\vec{Q}$

(B) $\vec{S} = (1-b^2)\vec{P} + b\vec{Q}$

(C) $\vec{S} = (1-b)\vec{P} + b\vec{Q}$

(D) $\vec{S} = (b-1)\vec{P} + b\vec{Q}$



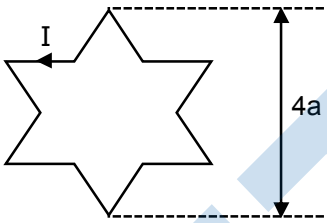
**Ans.** C

**Sol.** $\vec{R} = \vec{Q} - \vec{P}$

$\vec{S} = \vec{P} + b\vec{R} \quad (\because b\vec{R} = b|\vec{R}|\hat{R})$

$S = \vec{P} + b(Q - \vec{P}) = b\vec{Q} + \vec{P}(1-b)$

**3.** A symmetric star shaped conducting wire loop is carrying a steady state current I as shown in the figure. The distance between the diametrically opposite vertices of the star is 4a. The magnitude of the magnetic field at the center of the loop is




(A) $\frac{\mu_0 I}{4\pi a} 6[\sqrt{3} - 1]$

(B) $\frac{\mu_0 I}{4\pi a} 3[\sqrt{3} - 1]$

(C) $\frac{\mu_0 I}{4\pi a} 6[\sqrt{3} + 1]$

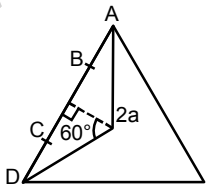
(D) $\frac{\mu_0 I}{4\pi a} 3[2 - \sqrt{3}]$

**Ans.** A

**Sol.** For our structure

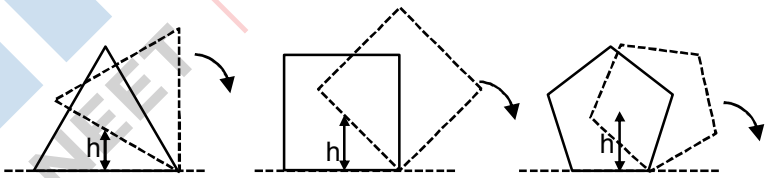
= 6 × (Magnetic field due to AD – Magnetic field due to BC)

$= = \frac{6\mu_0 I}{4\pi a}(\sqrt{3} - 1)$ $\qquad$ AD = $2\sqrt{3}a$




BC = $\frac{2}{\sqrt{3}}$

**4.** Consider regular polygons with number of sides n = 3, 4, 5 ..... as shown in the figure. The center of mass of all the polygons is at height h from the ground. They roll on a horizontal surface about the leading vertex without slipping and sliding as depicted. The maximum increase in height of the locus of the center of mass for each polygon is $\Delta$. Then $\Delta$ depends on n and h as

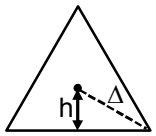


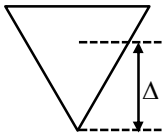

(A) $\Delta = h\sin\left(\frac{2\pi}{n}\right)$ $\qquad$ (B) $\Delta = h\sin^2\left(\frac{\pi}{n}\right)$

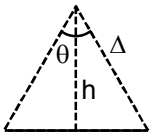
(C) $\Delta = h\tan^2\left(\frac{\pi}{2n}\right)$ $\qquad$ (D) $\Delta = h\left(\frac{1}{\cos\frac{\pi}{n}} - 1\right)$

**Ans.** D

**Sol.**




For general polygon $\theta = \frac{2\pi}{n}$

Maximum increase in height $\cos\frac{\theta}{2} = \frac{h}{\Delta}$

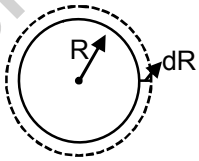
$= \Delta - h = h\left(\frac{1}{\cos(\theta/2)} - 1\right) = h\left(\frac{1}{\cos(\pi/2)} - 1\right)$ $\Delta = \frac{h}{\cos(\theta/2)}$

**5.** Consider an expanding sphere of instantaneous radius R whose total mass remains constant. The expansion is such that the instantaneous density $\rho$ remains uniform throughout the volume. The rate of fractional change in density $\left(\frac{1}{\rho}\frac{d\rho}{dt}\right)$ is constant. The velocity $v$ of any point on the surface of the expanding sphere is proportional to

(A) $R^{2/3}$ (B) R (C) $R^3$ (D) $\frac{1}{R}$

**Ans.** B

**Sol.** Let partical on surface moves dR in dt time.




$V = \frac{dR}{dt}$

$\frac{1}{\rho}\frac{d\rho}{dt} = c$ ($\because$ C = constant)

$\frac{d\rho}{dt} = \left(\frac{M}{(4/3)\pi}\right)\left(\frac{-3}{R^4}\right)\frac{dR}{dt}$

$= \frac{-3\rho dR}{dt}$ .....(ii)

by (i) & (ii)

$\frac{dR}{dt} \propto R$

**6.** A person measures the depth of a well by measuring the time interval between dropping a stone and receiving the sound of impact with the bottom of the well. The error in his measurement of time is $\delta T = 0.01$ seconds and he measures the depth of the well to be L = 20 meters. Take the acceleration due to gravity g = 10 $ms^{-2}$ and velocity of sound is 300 $ms^{-1}$. Then the fractional error in measurement, $\delta L/L$, is closest to

(A) 0.2% (B) 5% (C) 1% (D) 3%

**Ans.** C

**Sol.** Total time = time taken by stone to reach water + time taken by sound were

$$T = \sqrt{\frac{2L}{g}} + \frac{L}{300}$$

$$\delta T = \left(\frac{1}{\sqrt{2gL}} + \frac{1}{300}\right)\delta L = \left(\frac{1}{20} + \frac{1}{300}\right)\delta L$$

$$\frac{\delta L}{L} = \frac{0.01}{1 + \frac{1}{15}} \simeq \frac{0.01}{1}$$

$$\% \ \frac{\delta L}{L} = 1\%$$

**7.** A rocket is launched normal to the surface of the Earth, away from the sun, along the line joining the Sun and Earth. The sun is $3 \times 10^5$ times heavier than the Earth and is at a distance $2.5 \times 10^4$ times larger than the radius of the Earth. The escape velocity from Earth's gravitational field is $v_e = 11.2$ km $s^{-1}$. The minimum initial velocity ($v_s$) required for the rocket to be able to leave the Sun-Earth system is closest to (Ignore the rotation and revolution of the Earth and the presence of any other planet)

(A) $v_s = 62$ km $s^{-1}$ (B) $v_s = 42$ km $s^{-1}$ (C) $v_s = 72$ km $s^{-1}$ (D) $v_s = 22$ km $s^{-1}$

**Ans.** B

**Sol.** $M_S = 3 \times 10^5$ Me

Distance of Sun from Rocket (d) = $2.5 \times 10^4 R_e + R_e$

$$\frac{GM_e m}{R_e} M = \frac{1}{2} m V_e^2 \qquad \text{..... (i)}$$

For $V_s$

$$\frac{GMe}{R_e} m + \frac{GM_s}{d} m = \frac{1}{2} m v_s^2$$

$$V_s^2 - V_e^2 = \frac{3 \times 10^5}{2.5 \times 10^4} V_e^2$$

$V_s \simeq 42$

## SECTION 2 (Maximum Marks: 28)

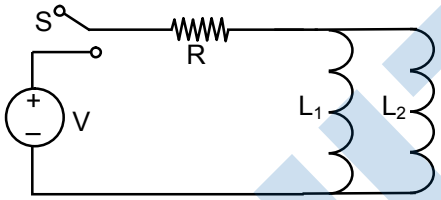
- This section contains **SEVEN** questions
- Each question has FOUR options [A], [B], [C] and [D]. ONE OR MORE THAN ONE of these four option is( are) correct.
- For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS.
- For each question, marks will be awarded in one of the following categories :

Full Marks : +4 If only the bubble(s) corresponding to all the correct option(s) is(are) darkened.

| | | | |
|---|---|---|---|
| Partial Marks | : | +1 | For darkening a bubble corresponding to each correct option, provided NO incorrect option is darkened. |
| Zero Marks | : | 0 | If none of the bubbles is darkened. |
| Negative Marks | : | –2 | In all other cases. |

- For example, if (A), (C) and (D) are all the correct options for a question, darkening all these three will get +4 marks; darkening only (A) and (D) will get +2 marks; and darkening (A) and (B) will get –2 marks, as a wrong option is also darkened.

**8.** A source of constant voltage V is connected to a resistance R and two ideal inductors $L_1$ and $L_2$ through a switch S as shown. There is no mutual inductance between the two inductors. The switch S is initially open. AT t = 0, the switch is closed and current begins to flow. Which of the following options is/are correct ?




(A) At t = 0, the current through the resistance R is $\frac{V}{R}$

(B) After a long time, the current through $L_2$ will be $\frac{V}{R}\frac{L_1}{L_1+L_2}$

(C) After a long time, the current through $L_1$ will be $\frac{V}{R}\frac{L_2}{L_1+L_2}$

(D) The ratio of the currents through $L_1$ and $L_2$ is fixed at all times (t > 0)

**Ans.** BCD

**Sol.** $V_{L_1} = V_{L_2}$

$$L_1\frac{dI_1}{dt} = L_2\frac{dI_2}{dt}$$

$$\frac{I_1}{I_2} = \frac{L_2}{L_1}$$

After a lng time circuit would be haves as pure resistive circuit and hence current would be $\frac{V}{R}$

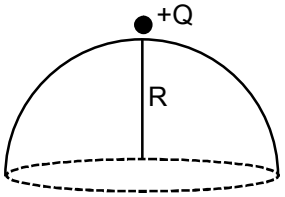
(A) $I_2 = \frac{V}{R}\times\frac{L_1}{L_1+L_2}$

(C) $I_1 = \frac{V}{R}\times\frac{L_2}{L_1+L_2}$

(D) for any instant of time current in induetor divide in inverse ratio of their inductance.

So the ratio of current through $L_1$ and $L_2$ is fixed at all times. (t > 0).

**9.** A point charge +Q is placed just outside an imaginary hemispherical surface of radius R as shown in the figure. Which of the following statements is/are correct ?




(A) The electric flux through the curved surface of the hemisphere is $-\frac{Q}{2\varepsilon_0}\left(1-\frac{1}{\sqrt{2}}\right)$

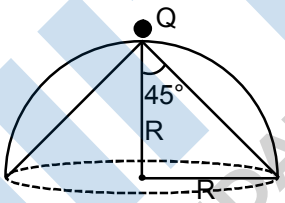
(B) The component of the electric field normal to the flat surface is constant over the surface

(C) Total flux through the curved and the flat surfaces is $\frac{Q}{\varepsilon_0}$

(D) The circumference of the flat surface is an equipotential

**Ans.** AD

**Sol.** $\phi$ circular = $\frac{Q}{4\pi\varepsilon_0}$ 2 $\pi$ (1 – cos 45°)



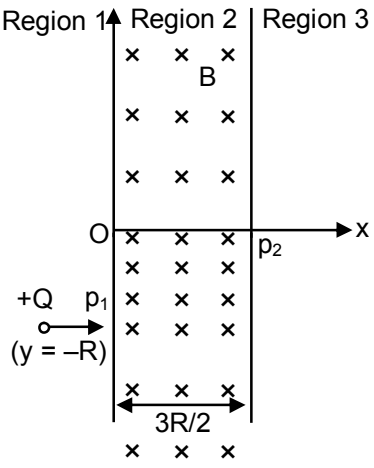

$= \frac{Q}{2\varepsilon_0} = 0$

$\phi$ cirular + $\phi$ curved = 0 $\Rightarrow$ $\phi$ curved = – $\phi$ ciruclar

$\Rightarrow \phi$ curved = $-\frac{Q}{2\varepsilon_0}\left(1-\frac{1}{\sqrt{2}}\right)$

(D) As all the point of circumtesence are at same distance from charge therefore potential is same at each point.

**10.** A uniform magnetic field B exists in the region between x = 0 and x = $\frac{3R}{2}$ (region 2 in the figure) pointing normally into the plane of the paper. A particle with charge +Q and momentum p directed along x-axis enters region 2 from region 1 at point $P_1$ (y = –R). Which of the following option(s) is/are correct ?



(A) For $B = \frac{8}{13}\frac{p}{QR}$, the particle will enter region 3 through the point $P_2$ on axis

(B) For $B > \frac{2}{3}\frac{p}{QR}$, the particle will re-enter region 1

(C) For a fixed B, particles of same charge Q and same velocity $\nu$, the distance between the point $P_1$ and the point of re-entry into region 1 is inversely proportional to the mass of the particle

(D) When the particle re-enters region 1 through the longest possible path in region 2, the magnitude of the change in its linear momentum between point $P_1$ and the farthest point from y-axis is $p/\sqrt{2}$

**Ans.** AB

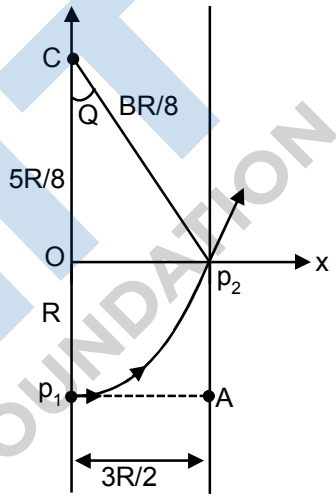
**Sol.** (A) Radius of curvature

$$r = \frac{mv}{qB} = \frac{p}{qB}$$

$$r = \frac{p \times 13QR}{Q8P} = \frac{13}{8}R$$

$$\text{If } B = \frac{8P}{13QR}$$

Where C is the centre of curvature



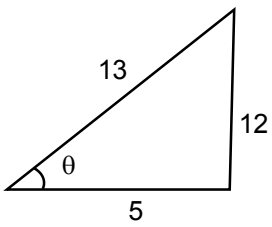

$$\sin\theta = \frac{oP_2}{CP_2} \Rightarrow \sin\theta = \frac{\frac{3R}{2}}{\frac{13R}{8}} = \frac{12}{13}$$

$AP_2 = (1 - \cos\theta)$

$$= \frac{13R}{8}\left(1 - \frac{5}{13}\right)$$

$$AP_2 = \frac{13R}{8} \times \frac{(8)}{13} = R$$

So it is clear that it passes through $P_2$

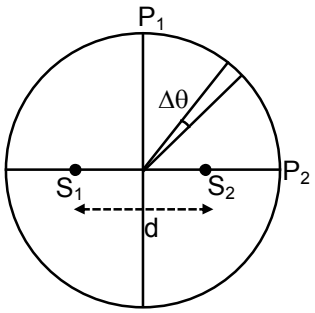



(B) Radius of curvature

$$r = \frac{P}{QB} = \frac{P}{Q\frac{2}{3}\frac{P}{QR}} = \frac{3R}{2}$$

i.e. particl will go back to region

**11.** Two coherent monochromatic point sources $S_1$ and $S_2$ of wavelength $\lambda$ = 600 nm are placed symmetrically on either side of the center of the circle as shown. The sources are separated by a distance d = 1.8 mm. This arrangement produces interference fringes visible as alternate bright and dark spots on the circumference of the circle. The angular separation between two consecutive bright spots is $\Delta\theta$. Which of the following options is/are correct ?




(A) The total number of fringes produced between $P_1$ and $P_2$ in first quadrant is close to 3000

(B) A dark spot will be formed at the point $P_2$

(C) At $P_2$ the order of the fringe will be maximum

(D) The angular separation between two consecutive bright spots decreases as we move from $P_1$ to $P_2$ along the first quadrant

**Ans.** AC

**Sol.** At $P_2 \Delta x = d = n\lambda$

$1.8 \times 10^{-3} = nx\ 600 \times 10^{-9}$

$n = \frac{18 \times 10^{-4}}{600 \times 10^{-9}}$

$= \frac{18 \times 10^{5}}{600} = 3 \times 10^{3} = 3000$

Total no. of fringes = 3000

(C) Order of finger is maximum at $P_2$ i.e. 3000.

(B) Condition satisties the constructive interforces hence the point $P_2$ is a bright spot. Hence option B is worng

(D) for any general angle $\theta$.

$\Delta x = d \cos\theta = n\lambda$

$\cos\theta = \frac{n\lambda}{d}$

$-\sin\theta\, d\theta = dn\frac{\lambda}{d}$

$D\theta = -\Delta n\frac{\lambda}{d\sin\theta}$

$\Delta\theta$ increases as $\theta$ decreases. so option D is also worng

**12.** The instantaneous voltages at three terminal marked X, Y and Z are given by

$V_X = V_0 \sin\omega t$,

$V_y = V_0 \sin\left(\omega t + \frac{2\pi}{3}\right)$ and

$V_Z = V_0 \sin\left(\omega t + \frac{4\pi}{3}\right)$

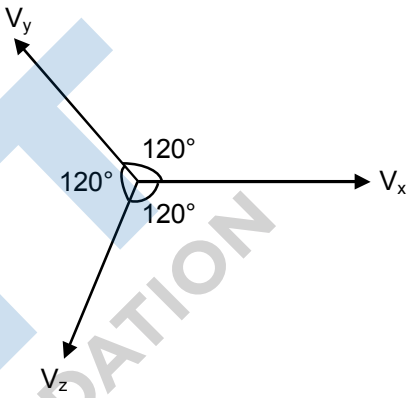
An ideal voltmeter is configured to read rms value of the potential difference between its termials. It is connected between X and Y and then between Y and Z. The reading(s) of the voltmeter will be connected between X and Y and then between Y and Z. The reading(s) of the voltmeter will be

(A) $V_{YZ}^{rms} = V_0\sqrt{\frac{1}{2}}$ (B) $V_{XY}^{rms} = V_0\sqrt{\frac{3}{2}}$ (C) $V_{XY}^{rms} = V_0$

(D) independent of the choice of the two terminals

**Ans.** BD

**Sol.** (B) $V_x = V_0 \sin \omega t$




$V_y = V_0 \sin\left(\omega t + \frac{2\pi}{3}\right)$

$V_z = V_0 \sin\left(\omega t + \frac{4\pi}{3}\right)$

$V_{xy} = V_y - v_x$

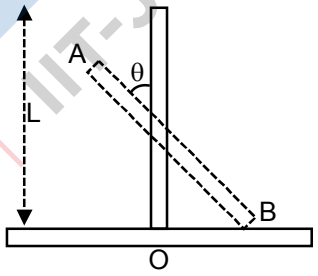
$V_{xy} = \sqrt{3}\ V_0$ (vectors cally)d

$(V_{rms})_{xy} = \frac{\sqrt{3}V_0}{\sqrt{2}}$

(D) $(V_{rms})_{xy} = (V_{rms})_{yz} = (V_{rms})_{xz}$

i.e. it does not depends upon the position where we are inserting the keys.

**13.** A rigid uniform bar AB of length L is slipping from its vertical position on a frictionless floor (as shown in the figure). At some instant of time, the angle made by the bar with the vertical is $\theta$. Which of the following statements about its motion is/are correct ?




(A) The trajectory of the point A is a parabola

(B) Instantaneous torque about the point in contact with the floor is proportional to $\sin\theta$

(C) The midpoint of the bar will fall vertically downward

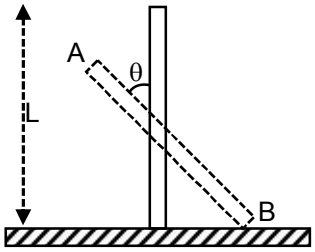
(D) When the bar makes an angle $\theta$ with the vertical, the displacement of its midpoint from the initial position is proportional to $(1 - \cos\theta)$

**Ans.** CD

**Sol.** (C) since the motion is only due to gravitational force and hece COM must move in vertically downwards direction.

(D) Initial height of COM = L/2

Final height of COM = $\frac{L}{2}\cos\theta$




height through which COM has fallen = $\frac{L}{2}-\frac{L}{2}\cos\theta$

$=\frac{L}{2}(1-\cos\theta)$

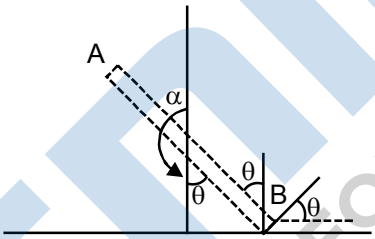
(A) $x=\frac{L}{2}\sin\theta$, $y = L\cos\theta$

$$\frac{x^2}{(L/2)}+\frac{y^2}{L^2}=1$$

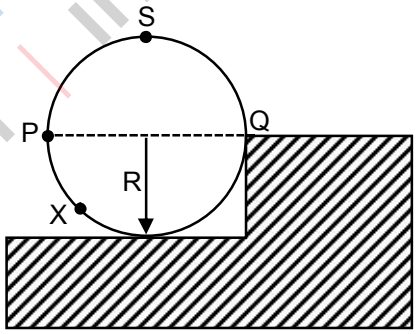
This equation is of ellipse.

Hence option A is wrong.

(B) The point about which equation is asking is crying on the Rod not on the ground so we have take the torque of pseudo force also which shows that torque about that point is not directly proportional t sin θ.




**14.** A wheel of radius R and mass M is placed at the bottom of a fixed step of height R as shown in the figure. A constant force is continuously applied on the surface of the wheel so that it just climbs the step without slipping. Consider the torque τ about an axis normal to the plane of the paper passing through the point  Which of the following options is/are correct ?




(A) If the force is applied tangentially at point S then $\tau \neq 0$ but the wheel never climbs the step

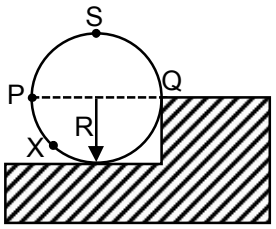
(B) If the force is applied normal to the circumference at point P then τ is zero

(C)  If the force is applied normal to the circumference at point X then τ is constant

(D) If the force is applied at point P tangentially then τ decreases continuously as the wheel climbs

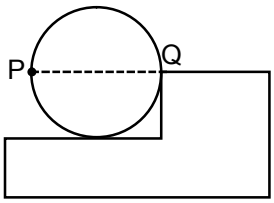
**Ans.** B

**Sol.** (A) A is wrong since as we apply force at point the sphere tries to rotate in clockwise direction due to which friction force at Q will acts in upwards direction as if force is increased that firction

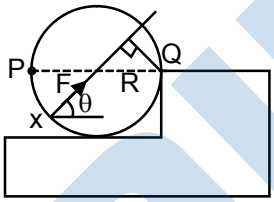



force wire lift up the sphere and the sphere climbs up the stairs.

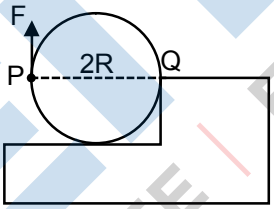
(B) If force is applied normal to circumference at P then the line of action of force passes through Q due to which torque is zero.




(C) $\tau_Q$ = F R sin $\theta$ it is a function of q so it is not constant hence wrong option.




(D)$\tau_Q$ = F × 2R = constant




So given statement is wrong

## SECTION 3 (Maximum Marks : 12)

- This section contains **TWO** paragraphs.
- Based on each paragraph, there are **TWO** questions.
- Each question has **FOUR options (A), (B), (C) and (D).ONLY ONE** of these four options is correct.
- For each question, darken the bubble corresponding to the correct option in the ORS
- For each question, marks will be awarded in one of the following categories :

| | | | |
|---|---|---|---|
| Full Marks | : | +3 | If only the bubble corresponding to the correct option is darkened. |
| Zero Marks | : | 0 | In all other cases. |

## PARAGRAPH-1

Consider a simple RC circuit as shown in figure 1.

**Process 1 :**In the circuit the switch S is closed at t = 0 and the capacitor if fully charged to voltage $V_0$ (i.e., charging continues for time T >> RC). In the process some dissipation ($E_D$) occurs across the resistance R. The amount of energy finally stored in the fully charged capacitor is $E_C$.

**Process 2 :** In a different process the voltage is first set to $\frac{V_0}{3}$ and maintained for a charging time T >>RC. Then the voltage is raised to $\frac{2V_0}{3}$ without discharging the capacitor and again maintained for a time T >> RC. The process is repeated one more time by raising the voltage to $V_0$ and the capacitor is charged to the same final voltage $V_0$ as in Process 1.

These two processes are depicted in figure 2.

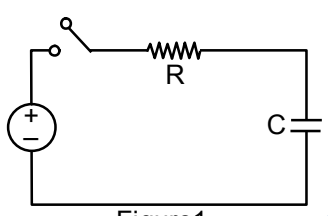
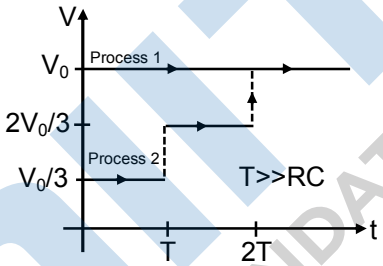


Figure1




**15** In Process 2, total energy dissipated across the resistance $E_D$ is:

(A) $E_D = 3\left(\frac{1}{2}CV_0^2\right)$ (B) $E_D = \frac{1}{2}CV_0^2$ (C) $E_D = 3CV_0^2$ (D) $E_D = \frac{1}{3}\left(\frac{1}{2}CV_0^2\right)$

**Ans.** D

**Sol.** $E_D$ = Work done by battery – energy stored in capacitor

$$= q_1\frac{V_0}{3} + q_3\frac{2V_0}{3} + q_3V_0 - \frac{1}{2}CV_0^2$$

$$q = \frac{CV_0}{3} \qquad q_2 = \frac{CV_0}{3} \qquad q_3 = \frac{CV_0}{3}$$

$$E_D = \frac{CV_0^2}{9} + \frac{2CV_0^2}{9} + \frac{3CV_0^2}{9} - \frac{1}{2}CV_0^2$$

$$E_D = \frac{1}{6}CV_0^2$$

$$E_D = \frac{1}{3}\left(\frac{1}{2}CV_0^2\right)$$

**16.** In Process 1, the energy stored in the capacitor $E_C$ and heat dissipated across resistance $E_D$ are related by :

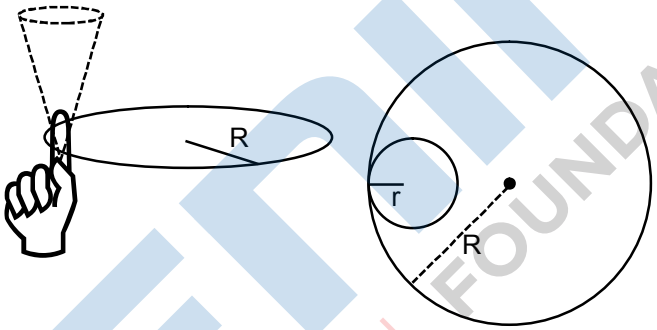
(A) $E_C = \frac{1}{2}E_D$ (B) $E_C = 2E_D$ (C) $E_C = E_D$ (D) $E_C = E_D \ln 2$

**Ans.** C

**Sol.** $E = E_D = \frac{1}{2} CV_0^2$

## PARAGRAPH-2

One twirls a circular ring (of mass M and radius R) near the tip of one's finger as shown in Figure 1. In the process the finger never loses contact with the inner rim of the ring. The finger traces out the surface of a cone, shown by the dotted line. The radius of the path traced out by the point where the ring and the figure is in contact is r. The finger rotates with an angular velocity $\omega_0$. The rotating ring rolls without slipping on the outside of a smaller circle described by the point where the ring and the finger is in contact (Figure 2). The coefficient of friction between the ring and the finger is m and the acceleration due to gravity is g.




**17.** The minimum value of $\omega_0$ belong which the ring will drop down is

(A) $\sqrt{\frac{2g}{\mu(R-r)}}$ (B) $\sqrt{\frac{g}{\mu(R-r)}}$ (C) $\sqrt{\frac{3g}{2\mu(R-r)}}$ (D) $\sqrt{\frac{g}{2\mu(R-r)}}$

**Ans.** **[B]**

**Sol.** $\mu N = mg$

$\mu m w_0^2 (R-r) = mg$

$w_0 = \sqrt{\frac{g}{\mu(R-r)}}$

**18.** The total kinetic energy of the ring is

(A) $\frac{3}{2}M\omega_0^2(R-r)^2$ (B) $\frac{1}{2}M\omega_0^2(R-r)^2$ (C) $M\omega_0^2(R-r)^2$ (D) $M\omega_0^2R^2$

**Ans.** **Bonus**

**Sol.** $(KE)_{Ring} = \frac{1}{2}mR^2w_0^2 + \frac{1}{2}m(R-r)^2W_0^2$

Which is not matching with any option

# PART B: CHEMISTRY

## SECTION 1 (Maximum Marks : 21)

- This section contains SEVEN questions.
- Each question has FOUR options (A), (B), (C) and (D). ONLY ONE of these four options is correct.
- For each question, darken the bubble corresponding to the correct option in the ORS.
- For each question, marks will be awarded in one of the following categories :

| | | |
|---|---|---|
| Full Marks | : +3 | If only the bubble corresponding to the correct option is darkened. |
| Zero Marks | : 0 | If none of the bubbles is darkened. |
| Negative Marks | : –1 | In all other cases. |

**19.** For the following cell,

$Zn(s) \mid ZnSO_4\ (a) \mid\mid CuSO_4\ (a) \mid Cu\ (s)$

when the concentration of $Zn^{2+}$ is 10 times the concentration of $Cu^{2+}$, the expression for $\Delta G$ (in J $mol^{-1}$) is

[F is Faraday constant; R is gas constant; T is temperature; E° (cell) = 1.1 V]

(A) 2.303 RT – 2.2 F (B) 2.303 RT + 1.1 F

(C) 1.1 F (D) –2.2 F

**Ans.** A

**Sol.** $Zn - 2e \longrightarrow Zn^{2+}$

$Cu^{2+} + 2e \longrightarrow Cu$

$Zn + Cu^{2+} \longrightarrow Zn^{2+} + Cu$

$$E_{cell} = E^\circ_{cell} - \frac{2.303RT}{nF}\log\frac{[Zn^{2+}]}{[Cu^{2+}]}$$

$$E_{cell} = 1.1 - \frac{2.303RT}{nF}\log 10$$

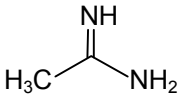
$$= 1.1 - \frac{2.303RT}{nF}$$

$\Delta G = -nFE_{cell}$

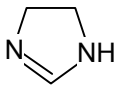
$$= -2F\left(1.1 - \frac{2.303RT}{2F}\right)$$

= 2.303RT – 2.2 F

**20.** The order of basicity among the following compounds is




(I)

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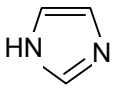
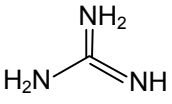


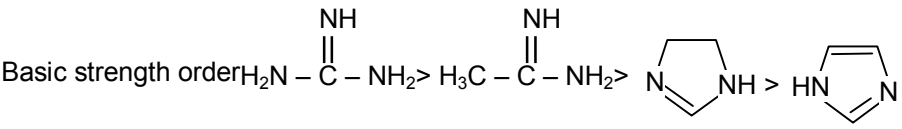

(III)




(IV)

(A) I > IV > III > II (B) II > I > IV > III (C) IV > I > II > III (D) IV > II > III > I

**Ans.** C

**Sol.** Basic strength order 

**21.** The standard state Gibbs free energies of formation of C(graphite) and C(diamond) at T = 298 K are

$\Delta_f G^\circ$ [C(graphite)] = 0 kJ $mol^{-1}$

$\Delta_f G^\circ$ [C(diamond)] = 2.9 kJ $mol^{-1}$

The standard state means that the pressure should be 1 bar, and substance should be pure at a given temperature. The conversion of graphite [C(graphite)] to diamond [C(diamond)] reduces its volume by 2 × $10^{-6}$ $m^3$ $mol^{-1}$. If C(graphite) is converted to C(diamond) isothermally at T = 298 K, the pressure at which C(graphite) is in equilibrium with C(diamond), is

[Useful information : 1 J = 1 kg $m^2$ $s^{-2}$; 1 Pa = 1 kg $m^{-1}$ $s^{-2}$; 1 bar = $10^5$ Pa]

(A) 14501 bar (B) 29001 bar (C) 1450 bar (D) 58001 bar

**Ans.** A

**Sol.** $\Delta G = PdV$

$\left[\Delta_f G^\circ_{(dimond)} - \Delta_f G^\circ_{(graphite)}\right] = PdV$

$2.9 \times 10^3$ J $mol^{-1}$ = P × 2 × $10^{-6}$ $m^3$ $mol^{-1}$

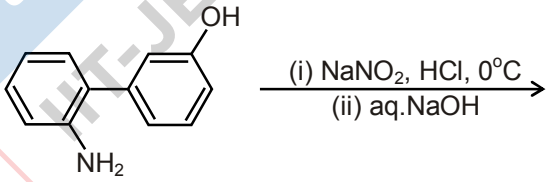
P = 1.45 ×$10^9$ Pa

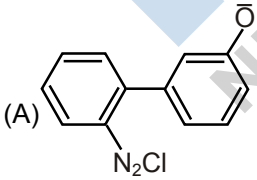
P = 1.45 ×$10^9$× $10^{-5}$ bar

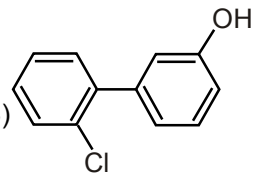
P = 1.45 × $10^4$ bar

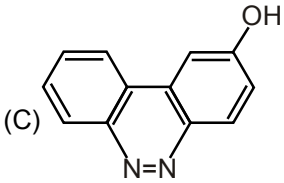
P = 14500 bar

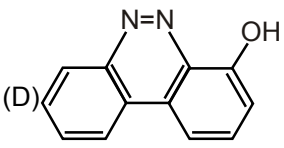
**22.** The major product of the following reaction is




(A) 

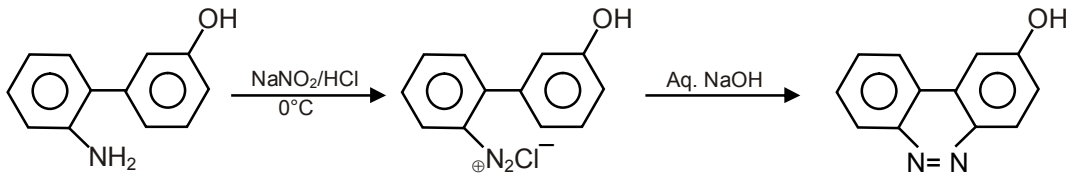

(B) 


(C) 


(D) 


**Ans.** C

**Sol.**




**23.** The order of the oxidation state of the phosphorus atom in $H_3PO_2$, $H_3PO_4$, $H_3PO_3$ and $H_4P_2O_6$ is

(A) $H_3PO_4 > H_3PO_2 > H_3PO_3 > H_4P_2O_6$ (B) $H_3PO_3 > H_3PO_2 > H_3PO_4 > H_4P_2O_6$

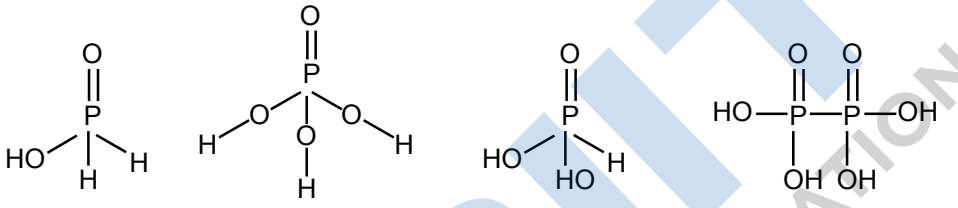
(C) $H_3PO_2 > H_3PO_3 > H_4P_2O_6 > H_3PO_4$ (D) $H_3PO_4 > H_4P_2O_6 > H_3PO_3 > H_3PO_2$

**Ans.** D

**Sol.** $H_3PO_2$ $H_3PO_4$ $H_3PO_3$ $H_4P_2O_6$

$4(+1) + 2x + 6(-2) = 0$

$2x = 8 \Rightarrow x = +4$



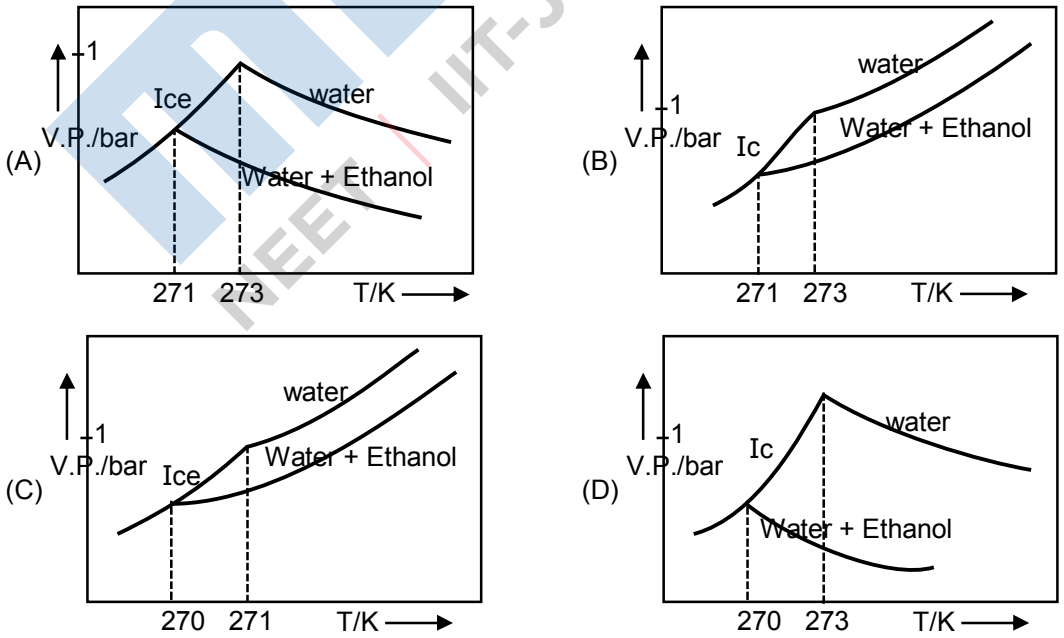
O.S. = +1 O.S. = +5 O.S. = +3 O.S. = +4

$H_3PO_4 > H_4P_2O_6 > H_3PO_3 > H_3PO_2$

**24.** Pure water freezes at 273 K and 1 bar. The addition of 34.5 g of ethanol to 500 g of water changes the freezing point of the solution. Use the freezing point depression constant of water as 2 K kg $mol^{-1}$. The figures shown below represent plots of vapour pressure (V.P.) versus temperature (T).

[molecular weight of ethanol is 46 g $mol^{-1}$]

Among the following, the option representing change in the freezing point is




**Ans.** C

**Sol.** $\Delta T_f = K_f \times m$

$= 2 \times \frac{34.5 \times 2}{46}$

= 2 × 1.5

= 3

Freezing point of ethanol + water mixture = 273- 3 = 270

**25.** Which of the following combination will produce $H_2$ gas?

(A) Au metal and NaCN (aq) in the presence of air

(B) Cu metal and conc. $HNO_3$

(C) Fe metal and conc. $HNO_3$

(D) Zn metal and NaOH (a)

**Ans.** D

**Sol.** $Au + NaCN(aq) \xrightarrow[\text{Cyanide forest}]{\text{MAC Arthur}} Na[Au(CN)_2] + 4NaOH$

$Fe + \text{conc. } HNO3 \longrightarrow Fe(NO_3)_3 + NO_2$

$Zn + NaOH(aq) \longrightarrow H_2\uparrow + Na_2ZnO_2$

## SECTION 2 (Maximum Marks : 28)

- This section contains **SEVEN**questions.
- Each question has **FOUR options (A), (B), (C) and (D). ONE OR MORE THAN ONE** of these four options is(are) correct.
- For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS.
- For each question, marks will be awarded in one of the following categories :

| | | | |
|---|---|---|---|
| Full Marks | : | +4 | If only the bubble(s) corresponding to all the correct option(s) is(are) darkened. |
| Partial Marks | : | +1 | For darkening a bubble corresponding to each correct option, provided NO incorrect option is darkened. |
| Zero Marks | : | 0 | If none of the bubbles is darkened. |
| Negative Marks | : | –2 | In all other cases. |

- For example, if (A), (C) and (D) are all the correct options for a question, darkening all these three will get +4 marks; darkening only (A) and (D) will get +2 marks; and darkening (A) and (B) will get –2 marks, as a wrong option is also darkened.

**26.** The option(s) with only amphoteric oxide is(are)

(A) $Cr_2O_3$, BeO, SnO, $SnO_2$ (B) $Cr_2O_3$, CrO, SnO, PbO

(C) ZnO, $Al_2O_3$, PbO, $PbO_2$ (D) NO, $B_2O_3$, PbO, $SnO_2$

**Ans.** AC

**Sol.** [A] $\rightarrow Cr_2O_3$, BeO, SnO, $SnO_2$

[C] $\rightarrow$ ZnO, $Al_2O_3$, PbO, $PbO_2$

are amphoteric

**27.** The correct statement(s) about surface properties is(are)

(A) Cloud is an emulsion type of colloid in which liquid is dispersed phase and gas is dispersion medium

(B) Adsorption is accompanied by decrease in enthalpy and decrease in entropy of the system

(C) The critical temperatures of ethane and nitrogen are 563 K and 126 K, respectively. The adsorption of ethane will be more than that of nitrogen on same amount of activated charcoal at a given temperature

(D) Brownian motion of colloidal particles does not depend on the size of the particles but depends on viscosity of the solution

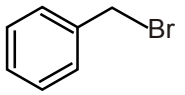
**Ans.** BC

**Sol.** In adsorption process both $\Delta H \& \Delta S$ is – ve. Higher the critical temperature of a gas higher the extent ofadsorption.

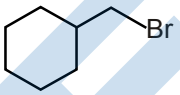
Cloud is not an emulsion.

Brownian motion depends on the size of the particles.

**28.** For the following compounds, the correct statement(s) with respect to nucleophilic substitution reactions is(are)




(I)

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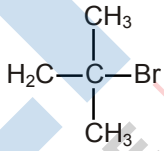
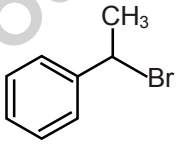



(III)




(IV)

(A) I and III follow $S_N1$ mechanism

(B) Compound IV undergoes inversion of configuration

(C) I and II follow $S_N2$ mechanism

(D) The order of reactivity for I, III and IV is : IV > I > III

**Ans.** ABC or ABCD

**Sol.** Benzylic and 3° halides both follow $S_N1$ mechanism.

Benzylic and 1° halides both follow $S_N2$ mechanism.

Benzylic 2° halides can undergo inversion of configuration.

The order of reactivity would be IV > I > III if both $S_N1$ and $S_N2$ are considered suitably for substrates.

**29.** Among the following, the correct statement(s) is(are)

(A) $BH_3$ has the three-centre two-electron bonds in its dimeric structure.

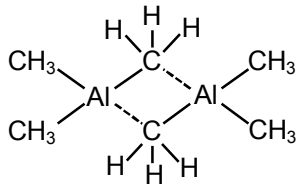
(B) $Al(CH_3)_3$ has the three-centre two-electron bonds in its dimeric structure.

(C) $AlCl_3$ has the three-centre two-electron bonds in its dimeric structure.

(D) The Lewis acidity of $BCl_3$ is greater than that of $AlCl_3$.

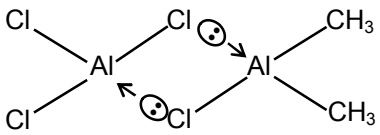
**Ans.** ABD

**Sol.** $2Al(CH_3)_3 \xrightarrow{\text{dimer}} Al_2(CH_3)_6$



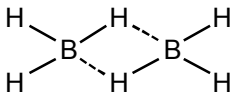
**3C – 2e⁻ bond**

$2\ AlCl_3 \xrightarrow{\text{dimer}} Al_2Cl_6$



**3C–4e⁻ bond**

$2\ BH_3 \xrightarrow{\text{dimer}} B_2H_6$



Lewis acidic character

Due to more electronegativity of B it has more lewis acidic character.

**30.** In a bimolecular reaction, the steric factor P was experimentally determined to be 4.5. The correct option(s) among the following is(are)

(A) Experimentally determined value of frequency factor is higher than that predicted by Arrhenius equation.

(B) The value of frequency factor predicted by Arrhenius equation is higher than that determined experimentally.

(C) The activation energy of the reaction is unaffected by the value of the steric factor.

(D) Since P = 4.5, the reaction will not proceed unless an effective catalyst is used.

**Ans.** AC

**Sol.** (A) Experimentally determined value of frequency factor is higher than that predicted by Arrhenius equation.

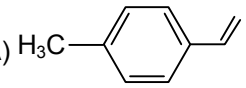
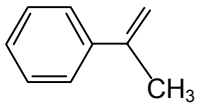
(C) The activation energy of the reaction is unaffected by the value of the steric factor.

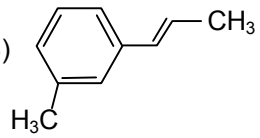
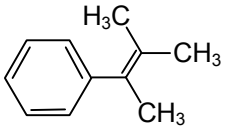
**31.** Compounds P and R upon ozonolysis produce Q and S, respectively. The molecular formula of Q and S is $C_8H_8O$. Q undergoes Cannizzaro reaction but not haloform reaction, whereas S undergoes haloform reaction but not Cannizzaro reaction.

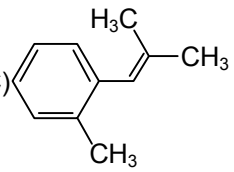
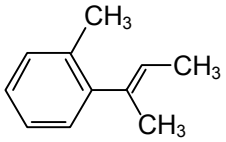
(i) $P \xrightarrow[\text{ii. } Zn/H_2O]{\text{i. } O_3/CH_2Cl_2} \underset{(C_8H_8O)}{Q}$

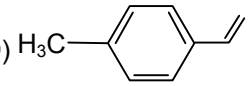
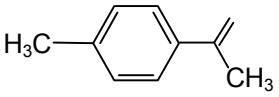
(ii) $R \xrightarrow[\text{ii. } Zn/H_2O]{\text{i. } O_3/CH_2Cl_2} \underset{(C_8H_8O)}{S}$

The option(s)with suitable combination ofPandR, respectively, is(are)

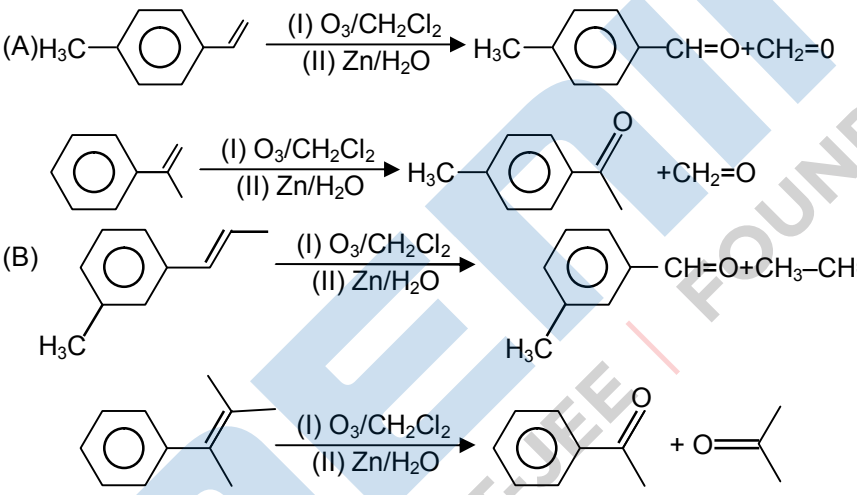
(A)  and 

(B)  and 

(C)  and 

(D)  and 

**Ans.** AB

**Sol.** 

Products formed by reactant C and D options do not have formula $C_8H_8O$

**32.** For a reaction taking place in a container in equilibrium with its surroundings, the effect of temperature on its equilibrium constant K in terms of change in entropy is described by

(A) With increase in temperature, the value of K for exothermic reaction decreases because the entropy change of the system is positive.

(B) With increase in temperature, the value of K for endothermic reaction increases because unfavourable change in entropy of the surroundings decreases

(C) With increase in temperature, the value of K for endothermic reaction increases because the entropy change of the system is negative.

(D) With increase in temperature, the value of K for exothermic reaction decreases because favourable change in entropy of the surroundings decreases.

**Ans.** BD

**Sol.** [B] With increase in temperature, the value of K for endothermic reaction increases because unfavourablechange in entropy of the surroundings decreases

[D] With increase in temperature, the value of K for exothermic reaction decreases because favourablechange in entropy of the surrounding decreases

## SECTION 3 (Maximum Marks : 12)

This section contains TWO paragraphs.

Based on each paragraph, there are TWO questions.

Each question has FOUR options (A), (B), (C) and (D). ONLY ONE of these four options is correct.

For each question, darken the bubble corresponding to the correct option in the ORS

For each question, marks will be awarded in one of the following categories :

| | | | |
|---|---|---|---|
| Full Marks | : | +3 | If only the bubble corresponding to the correct option is darkened. |
| Zero Marks | : | 0 | In all other cases. |

### PARAGRAPH 1

Upon heating $KClO_3$ in the presence of catalytic amount of $MnO_2$, a gas W is formed. Excess amount of W reacts with white phosphorus to give X. The reaction of X with pure $HNO_3$ gives Y and Z.

**33.** W and X are, respectively

(A) $O_2$ and $P_4O_{10}$ (B) $O_2$ and $P_4O_6$ (C) $O_3$ and $P_4O_6$ (D) $O_3$ and $P_4O_{10}$

**Ans.** A

**34.** Y and Z are, respectively

(A) $N_2O_5$ and $HPO_3$ (B) $N_2O_4$ and $HPO_3$ (C) $N_2O_4$ and $H_3PO_3$ (D) $N_2O_3$ and $H_3PO_4$

**Ans.** A

**Sol.**

$$KClO_3 \xrightarrow{MnO_2} KCl + \frac{3}{2}O_2\ [W]$$

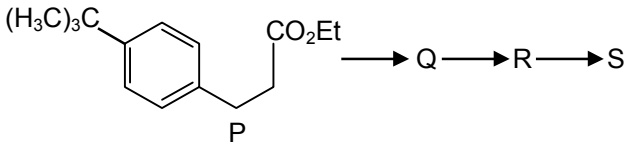
$$P_4 + \underset{(\text{excess amount})}{O_2} \rightarrow \underset{[X]}{P_4O_{10}}$$

$$P_4O_{10} + HNO_3 \rightarrow \underset{[Y]}{N_2O_5} + \underset{[Z]}{HPO_3}$$

### PARAGRAPH 2

The reaction of compound P with $CH_3MgBr$ (excess) in $(C_2H_5)_2O$ followed by addition of $H_2O$ gives The compound Q on treatment with $H_2SO_4$ at 0°C gives R. The reaction of R with $CH_3COCl$ in the presence of anhydrous $AlCl_3$ in $CH_2Cl_2$ followed by treatment with $H_2O$ produces compound S.

[Et in compound P is ethyl group]



**35.** The reactions, Q to R and R to S, are

(A) Dehydration and Friedel-Crafts acylation

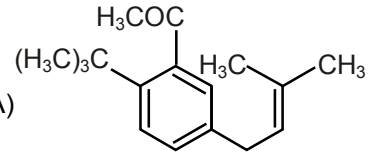
(B) Friedel-Crafts alkylation and Friedel-Crafts acylation

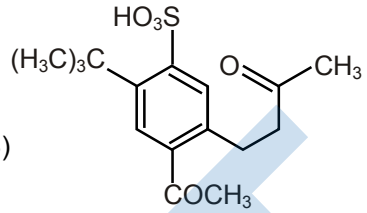
(C) Friedel-Crafts alkylation, dehydration and Friedel-Crafts acylation

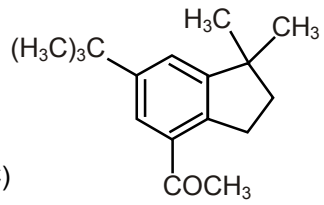
(D) Aromatic sulfonation and Friedel-Crafts acylation

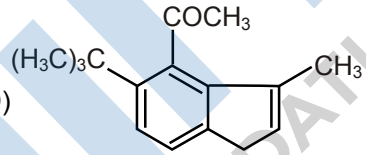
**Ans.** B

**36.** The product S is

(A) 

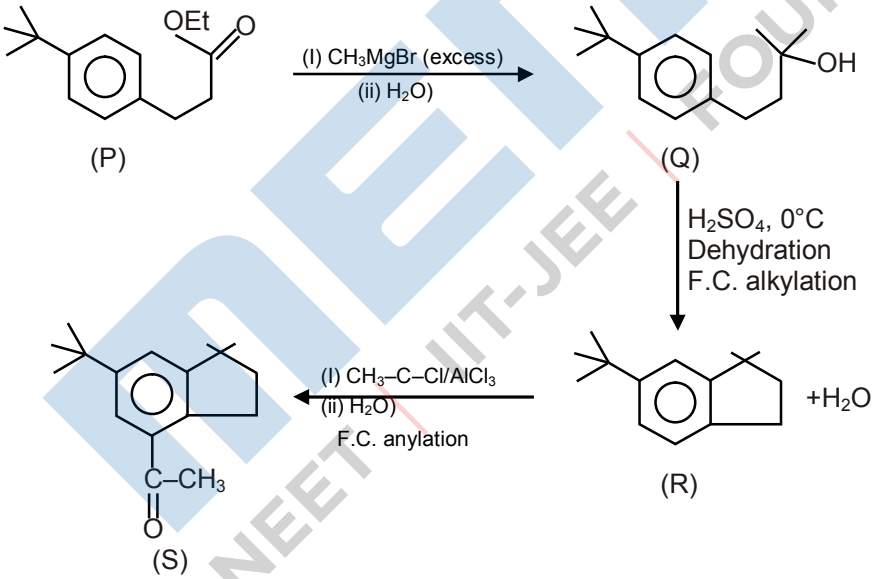
(B) 

(C) 

(D) 

**Ans.** C

**Sol.**



# PART C: MATHEMATICS

## SECTION 1 (Maximum Marks: 21)

- This section contains SEVEN questions
- Each question has FOUR options [A], [B], [C] and [D]. ONLY ONE of these four options is correct
- For each question, darken the bubble corresponding to the correct option in the ORS.
- For each question, marks will be awarded in one of the following categories :

| | | | |
|---|---|---|---|
| Full Marks | : | +3 | If only the bubble corresponding to the correct option is darkened. |
| Zero Marks | : | 0 | If none of the bubbles is darkened. |
| Negative Marks | : | –1 | In all other cases. |

**37.** Three randomly chosen non-negative integers x, y and z are found to satisfy the equation x + y + z = 10. Then the probability that z is even, is

(A) $\frac{5}{11}$ (B) $\frac{1}{2}$ (C) $\frac{6}{11}$ (D) $\frac{36}{55}$

**Ans.** C

**Sol.** x + y + z = 10

Total non-negative solution = ${}^{10+3-1}C_{3-1} = {}^{12}C_2 = \frac{12\times11}{2} = 66$

If z is even, then x + y = 10 or 8 or 6 or 4 or 2 or 0

$\therefore$ Number of solutions = ${}^{11}C_1 + {}^{9}C_1 + {}^{7}C_1 + {}^{5}C_1 + {}^{3}C_1 + {}^{1}C_1 = 11 + 9 + 7 + 5 + 3 + 1 = 36$

$\therefore$ Probability = $\frac{36}{66} = \frac{6}{11}$.

**38.** If f : R $\rightarrow$ R is a twice differentiable function such that f "(x) > 0 for all x $\in$ R, and $f\left(\frac{1}{2}\right) = \frac{1}{2}$, f(1) = 1, then

(A) f '(1) > 1 (B) f '(1) $\leq$ 0 (C) $\frac{1}{2} <$ f '(1) $\leq 1$ (D) $0 <$ f '(1) $\leq \frac{1}{2}$

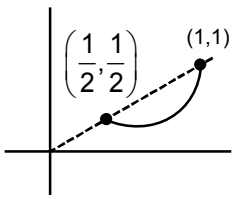
**Ans.** A

**Sol.** f '(c) = $\frac{f(1)-f\left(\frac{1}{2}\right)}{1-\frac{1}{2}} = 1$

For some c $\in \left(\frac{1}{2}, 1\right)$

$\because$ f '(x) is increasing.

$\therefore$ f '(1) > 1.



**39.** The equation of the plane passing through the point (1, 1, 1) and perpendicular to the planes $2x + y - 2z = 5$ and $3x - 6y - 2z = 7$, is

(A) $-14x + 2y + 15z = 3$ (B) $14x + 2y - 15z = 1$

(C) $14x + 2y + 15z = 31$ (D) $14x - 2y + 15z = 27$

**Ans.** C

**Sol.** $\because$ Plane is perpendicular to both planes.

$$\therefore \text{ normal of required plane, } \vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -2 \\ 3 & -6 & -2 \end{vmatrix} = -14\hat{i} - 2\hat{j} - 15\hat{k}$$

$\therefore$ Plane will be $14(x-1) + 2(y-1) + 15(z-1) = 0 \Rightarrow 14x + 2y + 15z = 31$. **Ans.**]

**40.** Let O be the origin and let PQR be an arbitrary triangle. The point S is such that

$$\overrightarrow{OP}\cdot\overrightarrow{OQ} + \overrightarrow{OR}\cdot\overrightarrow{OS} = \overrightarrow{OR}\cdot\overrightarrow{OP} + \overrightarrow{OQ}\cdot\overrightarrow{OS} = \overrightarrow{OQ}\cdot\overrightarrow{OR} + \overrightarrow{OP}\cdot\overrightarrow{OS}.$$

Then the triangle PQR has S as its

(A) circumcentre (B) orthocenter (C) incentre (D) centroid

**Ans.** B

**Sol.** Let p.v.s of P, Q, R & S be $\vec{p}, \vec{q}, \vec{r}$ & $\vec{s}$ $\vec{p}\cdot\vec{q} + \vec{r}\cdot\vec{s} = \vec{r}\cdot\vec{p} + \vec{q}\cdot\vec{s}$

$\therefore \vec{p}\cdot\vec{q} + \vec{r}\cdot\vec{s} = \vec{r}\cdot\vec{p} + \vec{q}\cdot\vec{s}$

$\Rightarrow \vec{p}\cdot(\vec{q} - \vec{r}) + \vec{s}\cdot(\vec{r} - \vec{q}) = 0 \Rightarrow (\vec{p} - \vec{s})\cdot(\vec{q} - \vec{r}) = 0$

Similarly $(\vec{q} - \vec{s})\cdot(\vec{r} - \vec{p}) = 0$

$\therefore$ Point S ( $\vec{s}$ ) is orthocentre of $\Delta PQR$.

**41.** How many 3 × 3 matrices M with entries from {0, 1, 2} are there, for which the sum of the diagonal entries of $M^TM$ is 5?

(A) 126 (B) 198 (C) 162 (D) 135

**Ans.** B

**Sol.** Let $M = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$

$$\therefore M^TM = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} = \begin{bmatrix} a_1^2 + a_2^2 + a_3^2 & - & - \\ - & b_1^2 + b_2^2 + b_3^2 & - \\ - & - & c_1^2 + c_2^2 + c_3^2 \end{bmatrix}$$

$$\therefore \text{tr}(M^TM) = \sum a_i^2 + b_i^2 + c_i^2 = 5$$

$\therefore$ 7 out of 9 should be 0 and one 1 and one 2.

$$\therefore \text{ Number of ways } = \frac{9!}{7!} = 9 \times 8 = 72.$$

or 4 zeros and 5 one = $\frac{9!}{4! \cdot 5!}$ = 126

$\therefore$ Total number of ways = 198.

**42.** Let S = {1, 2, 3, ……., 9}. For k = 1, 2, …, 5, let $N_k$ be the number of subsets of S, each containing five elements out of which exactly k are odd. Then $N_1 + N_2 + N_3 + N_4 + N_5$ =

(A) 210 (B) 252 (C) 125 (D) 126

**Ans.** D

**Sol.**

| 5 odd | + | 4 even |
|---|---|---|
| 1 | | 4 |
| 2 | | 3 |
| 3 | | 2 |
| 4 | | 1 |
| 5 | | 0 |

Same as taking any 5 out of 9 = ${}^9C_5 = {}^9C_4$ = 126. **Ans.**]

**43.** If y = y(x) satisfies the differential equation $8\sqrt{x}\left(\sqrt{9+\sqrt{x}}\right)dy = \left(\sqrt{4+\sqrt{9+\sqrt{x}}}\right)^{-1} dx$ , x > 0and y(0) = $\sqrt{7}$ , then y(256) =

(A) 16 (B) 80 (C) 3 (D) 9

**Ans.** C

**Sol.** $\int dy = \int \frac{dx}{8\sqrt{x}\left(\sqrt{9+\sqrt{x}}\right)\left(\sqrt{4+\sqrt{9+\sqrt{x}}}\right)}$

Put $\sqrt{4+\sqrt{9+\sqrt{x}}} = t \Rightarrow \frac{dx}{8\sqrt{x}\left(9+\sqrt{x}\right)\left(\sqrt{4+\sqrt{9+\sqrt{x}}}\right)} = dt\, t$

$\Rightarrow y = \sqrt{4+\sqrt{9+\sqrt{x}}} + C$

$y(0) = \sqrt{7} = \sqrt{7} + C \Rightarrow C = 0$

$\therefore y(256) = \sqrt{4+\sqrt{9+16}} = \sqrt{4+5} = 3$ .

## SECTION 2 ((Maximum Marks: 28)

- This section contains SEVEN questions
- Each question has FOUR options [A], [B], [C] and [D]. ONE OR MORE THAN ONE of these four option is( are) correct.
- For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS.
- For each question, marks will be awarded in one of the following categories :

Full Marks : +4 If only the bubble(s) corresponding to all the correct option(s) is(are) darkened.

| | | | |
|---|---|---|---|
| Partial Marks | : | +1 | For darkening a bubble corresponding to each correct option, provided NO incorrect option is darkened. |
| Zero Marks | : | 0 | If none of the bubbles is darkened. |
| Negative Marks | : | –2 | In all other cases. |

- For example, if (A), (C) and (D) are all the correct options for a question, darkening all these three will result in +4 marks; darkening only (A) and (D) will result in +2 marks; and darkening (A) and (B) will result in –2 marks, as a wrong option is also darkened.

**44.** Let $f(x) = \frac{1-x\left(1+|1-x|\right)}{|1-x|}\cos\left(\frac{1}{1-x}\right)$ for $x \neq 1$. Then

(A) $\lim\limits_{x\to1^-} f(x)$ does not exist

(B) $\lim\limits_{x\to1^+} f(x)$ does not exist

(C) $\lim\limits_{x\to1^-} f(x) = 0$

(D) $\lim\limits_{x\to1^+} f(x) = 0$

**Ans.** BC

**Sol.** $f(x) = \frac{1-x\left(1+|1-x|\right)}{|1-x|}\cos\left(\frac{1}{1-x}\right)$ for $x \neq 1$

$$\lim_{x\to1^-} f(x) = \lim_{x\to1^-} \frac{1-x(1+1-x)}{|1-x|}\cos\left(\frac{1}{1-x}\right) = \lim_{x\to1^-} \frac{1-2x+x^2}{1-x}\cos\left(\frac{1}{1-x}\right)$$

$$\lim_{x\to1^-} f(x) = \lim_{x\to1^-} \frac{(1-x)^2}{1-x}\cos\left(\frac{1}{1-x}\right) = 0$$

$$\lim_{x\to1^+} f(x) = \lim_{x\to1^+} \frac{1-x(1-1+x)}{|1-x|}\cos\left(\frac{1}{1-x}\right) = \lim_{x\to1^+} \frac{1-x^2}{x-1}\cos\left(\frac{1}{1-x}\right)$$

$$\lim_{x\to1^+} f(x) = \lim_{x\to1^+} -(1+x)\cdot\cos\left(\frac{1}{1-x}\right) = \text{Does not exist.}$$

**45.** Let $\alpha$ and $\beta$ be non-zero real numbers such that $2(\cos\beta - \cos\alpha) + \cos\alpha\cos\beta = 1$. Then which of the following is/are true?

(A) $\sqrt{3}\tan\left(\frac{\alpha}{2}\right) + \tan\left(\frac{\beta}{2}\right) = 0$

(B) $\sqrt{3}\tan\left(\frac{\alpha}{2}\right) - \tan\left(\frac{\beta}{2}\right) = 0$

(C) $\tan\left(\frac{\alpha}{2}\right) + \sqrt{3}\tan\left(\frac{\beta}{2}\right) = 0$

(D) $\tan\left(\frac{\alpha}{2}\right) - \sqrt{3}\tan\left(\frac{\beta}{2}\right) = 0$

**Ans.** CD

**Sol.** $\frac{2}{1} = \frac{1-\cos\alpha\cos\beta}{\cos\beta-\cos\alpha}$

Using C & D

$$\frac{2+1}{2-1} = \frac{1-\cos\alpha\cos\beta+\cos\beta-\cos\alpha}{1-\cos\alpha\cos\beta-\cos\beta+\cos\alpha}$$

$$\Rightarrow 3 = \frac{(1+\cos\beta)(1-\cos\alpha)}{(1-\cos\beta)(1+\cos\alpha)} \Rightarrow 3 = \frac{\tan^2\frac{\alpha}{2}}{\tan^2\frac{\beta}{2}} \Rightarrow \tan\frac{\alpha}{2} = \pm\sqrt{3}\tan\frac{\beta}{2}$$

$\Rightarrow \tan\frac{\alpha}{2} \mp \sqrt{3}\tan\frac{\beta}{2} = 0$ .

**46.** If $f(x) = \begin{vmatrix} \cos 2x & \cos 2x & \sin 2x \\ -\cos x & \cos x & -\sin x \\ \sin x & \sin x & \cos x \end{vmatrix}$, then

(A) f '(x) = 0 at exactly three points in $(-\pi, \pi)$

(B) f '(x) = 0 at more than three points in $(-\pi, \pi)$

(C) f(x) attains its maximum at x = 0

(D) f (x) attains its minimum at x = 0

**Ans.** BC

**Sol.** Applying $C_1 \to C_1 - C_2$, we get

$$f(x) = \begin{vmatrix} 0 & \cos 2x & \sin 2x \\ -2\cos x & \cos x & -\sin x \\ 0 & \sin x & \cos x \end{vmatrix} = 2\cos x (\cos 2x \cos x - \sin 2x \sin x)$$

$f(x) = 2\cos x \cos 3x = \cos 4x + \cos 2x$

$\therefore$ f(x) will take minimum at x = 0 and

f '(x) = – 4 sin 4x – 2sin 2x = – 2sin 2x (4cos 2x + 1) = 0 at 7 point where $2x \in (-2\pi, 2\pi)$

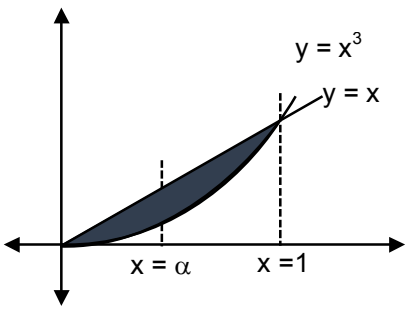
$\therefore$ More than three points.

**47.** If the line $x = \alpha$ divides the area of region $R = \{(x, y) \in R^2 : x^3 \le y \le x, 0 \le x \le 1\}$ into two equal parts, then

(A) $2\alpha^4 - 4\alpha^2 + 1 = 0$ (B) $\alpha^4 + 4\alpha^2 - 1 = 0$ (C) $0 < \alpha \le \frac{1}{2}$ (D) $\frac{1}{2} < \alpha < 1$

**Ans.** AD

**Sol.** $\because \int_0^\alpha (x - x^3)\,dx = \int_\alpha^1 (x - x^3)\,dx$




$\Rightarrow \left(\frac{\alpha^2}{2} - \frac{\alpha^4}{4}\right) = \left(\frac{1}{2} - \frac{1}{4}\right) - \left(\frac{\alpha^2}{2} - \frac{\alpha^2}{4}\right)$

$\Rightarrow \alpha^2 - \frac{\alpha^4}{2} = \frac{1}{4} \Rightarrow 2\alpha^4 - 4\alpha^2 + 1 = 0$

$\Rightarrow \alpha^2 = 1 \pm \frac{1}{\sqrt{2}}$

$0 < \alpha^2 < 1$

$\therefore \alpha^2 = 1 - \frac{1}{\sqrt{2}} \simeq 0.29$

$\therefore \alpha > 0.5 \Rightarrow \frac{1}{2} < \alpha < 1.$

**48.** If $I = \sum_{k=1}^{98} \int_{k}^{k+1} \frac{k+1}{x(x+1)} dx$, then

(A) $I < \frac{49}{50}$ (B) $I > \log_e 99$ (C) $I > \frac{49}{50}$ (D) $I < \log_e 99$

**Ans.** CD

**Sol.** $\sum_{k=1}^{98} \int_{k}^{k+1} \frac{k+1}{x(x+1)} dx$

In $(K, k+1)$ $\frac{k+1}{x(x+1)} < \frac{k+1}{x(k+1)}$ as $x + 1 \in (k+1, k+2)$

$\therefore \int_{k}^{k+1} \frac{k+1}{x(x+1)} dx < \int_{k}^{k+1} \frac{dx}{x} < \ln(k+1) - \ln k$

$\therefore \sum_{k=1}^{98} I < \sum_{k=1}^{98} \ln(k+1) - \ln k < \ln 99$

Again in $(k, k+1)$ where $k \in N$; $k > 1$

$\frac{k+1}{x(x+1)} \geq \frac{2}{x(x+1)} \geq \frac{2}{(x+1)^2}$

$\therefore I = \int_{k}^{k+1} \frac{2}{(x+1)^2} dx > \left(\frac{1}{x+1}\right)\Big]_{k+1}^{k} > 2\left(\frac{1}{k+1} - \frac{1}{k+2}\right)$

$\sum_{k=1}^{98} I > 2\left[\left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + ..... + \left(\frac{1}{99} - \frac{1}{100}\right)\right]$

$> 2\left[\frac{1}{2} - \frac{1}{100}\right] > \frac{49}{50}$ .....(2)

**49.** If $f : R \to R$ is a differentiable function such that $f'(x) > 2f(x)$ for all $x \in R$, and $f(0) = 1$, then

(A) $f(x)$ is decreasing in $(0, \infty)$ (B) $f'(x) < e^{2x}$ in $(0, \infty)$

(C) $f(x)$ is increasing in $(0, \infty)$ (D) $f(x) > e^{2x}$ in $(0, \infty)$

**Ans.** CD

**Sol.** $f'(x) > 2f(x)$, $f(0) = 1$

$f'(x) - 2f(x) > 0$

$\frac{d}{dx}\left(e^{-2x} f(x)\right) > 0$

$\therefore$ $g(x) = e^{-2x} f(x)$ is increasing function.

$g(0) = 1$

$\therefore$ $g(x) > 1 \ \forall \ x > 0$

$\Rightarrow f(x) > e^{2x} \forall \ x > 0$

$\therefore$ $f(x) > 0$

$\therefore$ f '(x) > 0

$\therefore$ f is increasing in (0, $\infty$).

**50.** If $g(x) = \int_{\sin x}^{\sin 2x} \sin^{-1}(t)\,dt$, then

(A) $g'\left(\frac{\pi}{2}\right) = -2\pi$ (B) $g'\left(\frac{-\pi}{2}\right) = -2\pi$ (C) $g'\left(\frac{-\pi}{2}\right) = 2\pi$ (D) $g'\left(\frac{\pi}{2}\right) = 2\pi$

**Ans.** BONUS

## SECTION 3 (Maximum Marks : 12)

This section contains **TWO** paragraphs.

Based on each paragraph, there are **TWO** questions.

Each question has FOUR options (A), (B), (C) and (D). **ONLY ONE** of these four options is correct.

For each question, darken the bubble corresponding to the correct option in the ORS

For each question, marks will be awarded in one of the following categories :

| | | | |
|---|---|---|---|
| Full Marks | : | +3 | If only the bubble corresponding to the correct option is darkened. |
| Zero Marks | : | 0 | In all other cases. |

### Paragraph for question 51 & 52

Let O be the origin, and $\overrightarrow{OX}, \overrightarrow{OY}, \overrightarrow{OZ}$ be three unit vectors in the directions of the sides $\overrightarrow{QR}, \overrightarrow{RP}, \overrightarrow{PQ}$, respectively, of a triangle PQR.

**51.** $\left|\overrightarrow{OX} \times \overrightarrow{OY}\right|$ =

(A) sin (P + R) (B) sin 2R (C) sin (Q + R) (D) sin (P + Q)

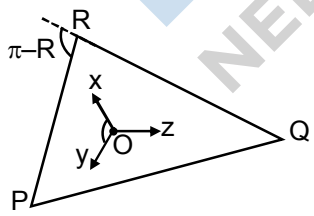
**52.** If the triangle PQR varies, then the minimum value of cos (P + Q) + cos (Q + R) + cos (R + P) is

(A) $\frac{-3}{2}$ (B) $\frac{5}{3}$ (C) $\frac{3}{2}$ (D) $\frac{-5}{3}$

**Sol.**

**51.** D

**52.** A




**(i)** $\left|\overrightarrow{OX} \times \overrightarrow{OY}\right|$ = sin (P + Q)

**(ii)** cos (P + Q) + cos (Q + R) + cos (R + P) = – (cos P + cos Q + cos R)

$\left|\overrightarrow{OX} + \overrightarrow{OY} + \overrightarrow{OZ}\right|^2 \geq 0$

$1 + 1 + 1 + 2\left(\cos(\pi - R) + \cos(\pi - P) + \cos(\pi - Q)\right) \geq 0$

$-(\cos P + \cos Q + \cos R) \geq \frac{-3}{2}$.

## Paragraph for question 53 & 54

Let p, q be integers and let $\alpha$, $\beta$ be the roots of the equation, $x^2 - x - 1 = 0$, where $\alpha \neq \beta$. For $n = 0, 1, 2, \ldots\ldots$, let $a_n = p\alpha^n + q\beta^n$.

Fact: If a and b are rational numbers and $a + b\sqrt{5} = 0$, then $a = 0 = b$.

**53.** If $a_4 = 28$, then $p + 2q =$

(A) 7 (B) 21 (C) 14 (D) 12

**54.** $a_{12} =$

(A) $a_{11} + a_{10}$ (B) $a_{11} - a_{10}$ (C) $a_{11} + 2a_{10}$ (D) $2a_{11} + a_{10}$

**Sol.**

**53.** D

**54.** A

**(i)** $x^2 - x - 1 = 0$

$x = \frac{1+\sqrt{5}}{2}$

$\alpha = \frac{1+\sqrt{5}}{2}, \beta = \frac{1-\sqrt{5}}{2}$

$a_4 = 28$

$$a_4 = p\left(\frac{1+\sqrt{5}}{2}\right)^4 + q\left(\frac{1-\sqrt{5}}{2}\right)^4 = 28$$

$$\Rightarrow \frac{1}{16}\left(p\left(56+24\sqrt{5}\right)+q\left(56-24\sqrt{5}\right)\right)=28$$

$$\Rightarrow \frac{1}{16}\left(p\left(56+24\sqrt{5}\right)+q\left(56-24\sqrt{5}\right)\right)=28$$

$\Rightarrow p - q = 0 \Rightarrow p = q$

$\Rightarrow \frac{56 \cdot 2p}{16} = 28 \Rightarrow p = 4 = q$

$\therefore p + 2q = 3p = 12$.

**(ii)** $a_{12} = p\alpha^{12} + q\beta^{12}$ .....(1)

$a_{10} = p\alpha^{10} + q\beta^{10}$ .....(2)

$a_{11} = p\alpha^{11} + q\beta^{11}$ .....(3)

$a_{10} + a_{11} = p\alpha^{10}(\alpha + 1) + q\beta^{10}(\beta + 1)$

We have, $a^2 = \alpha + 1$ and $b^2 = \beta + 1$

$\therefore a_{10} + a_{11} = p\alpha^{12} + q\beta^{12} = a_{12}$.